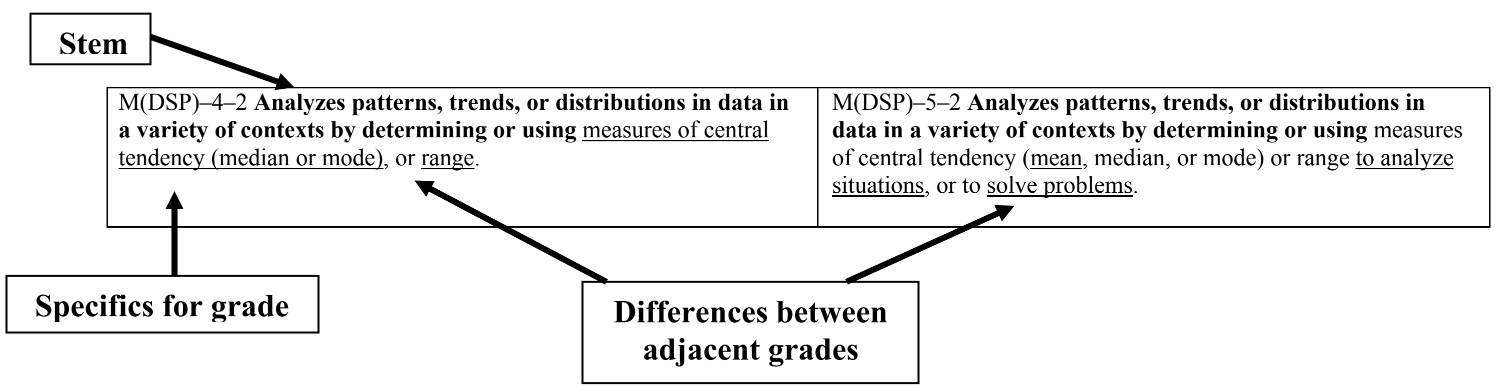
NECAP Mathematics Grade-Level Expectations May 11, 2004

The New England Common Assessment Program (NECAP) Mathematics GLEs have been developed as a means to identify the concepts and skills expected of all students for large-scale assessment of mathematics in grades 3–8; they are not intended to represent the full mathematics curriculum at each grade level, but are meant to capture concepts and skills related to “big ideas” of mathematics that can be assessed in an on-demand setting that focus the curriculum, but do not narrow the curriculum. Each partner state intends to develop a set of local GLEs to accompany these GLEs for local assessment purposes that includes the concepts and skills not easily assessable in an on-demand setting, and therefore not included in this set of GLEs.

The NECAP GLEs in this document can be interpreted as describing grade-level expectations for the end of the grade identified, or in the beginning of the next grade.

As you review the NECAP Mathematics Grade-Level Expectations the following are important to understand.

1) The NECAP GLEs are organized into four content strands: Number and Operations; Functions and Algebra; Data, Statistics, and Probability; and Geometry and Measurement.
2) Problem solving, reasoning, connections, and communication are embedded throughout this set of GLEs instead of as separate strands.
3) Each GLE includes a **bolded** statement called the “stem.” Each “stem” is the same or similar across the grades for a given GLE, and is meant to communicate the main curriculum and instructional focus of the GLE across the grades.
4) The unbolded text within a GLE indicates how the GLE is specified at a given grade level.
5) At each grade level differences from previous grades are underlined. (Note: Sometimes nothing is underlined within a GLE. In these situations examine other GLEs across the strand to identify the differences.)
6) Each GLE is coded for the content strand, grade level, and the GLE “stem” number (e.g., M(F&A)–6–3: The “M” stands for mathematics, the “F&A” stands for the functions and algebra strand, the “6” stands for grade 6, and the “3” stands for stem 3).
7) An empty cell means that the GLE “stem” will not be assessed at that grade on the state-level on-demand assessment, but is reserved for local curriculum and assessment.
8) Unless otherwise specified the number parameters for a given grade in M(N&O)–X–1 apply to all GLEs at that grade level.
9) Only number concepts identified at a grade level in the NECAP Numbers and Operations strand will be assessed and reported. However, all number concepts acquired up to a grade can be used in other content strands unless otherwise specified.
10) All the concepts and skills identified at a given grade level are “fair game” for assessment purposes. However, conjunctions in this document have specific meaning. The conjunction “and” separates parts of a GLE that will be assessed every year (to the extent possible), while the conjunction “or” separates parts of the GLE that may be assessed each year, but will be more likely to be assessed over several years. In some situations “or^{sc}” is used. While students will have choices on strategies they use or methods to communicate their thinking throughout the assessment, there are special cases that the New England partners thought it was necessary to communicate to the test developer that students should not be required to use a specific method (e.g., “…writes in words or^{sc} symbols…”).



Number and Operations

Grade 2	Grade 3	Grade 4
M(N&O)–2–1 **Demonstrates conceptual understanding of rational numbers with respect to:** **whole numbers** from 0 to 199 using place value, by applying the concepts of equivalency in composing or decomposing numbers (e.g., 34 = 17 + 17; 34 = 29 + 5); and in expanded notation (e.g., 141 = 1 hundred + 4 tens + 1 one or 141 = 100 + 40 + 1) **using models, explanations, or other representations**; and	M(N&O)–3–1 **Demonstrates conceptual understanding of rational numbers with respect to:** **whole numbers** from 0 to 999 through equivalency, composition, decomposition, or place value **using models, explanations, or other representations**; and	M(N&O)–4–1 **Demonstrates conceptual understanding of rational numbers with respect to:** **whole numbers** from 0 to 999,999 through equivalency, composition, decomposition, or place value **using models, explanations, or other representations**; and
positive fractional numbers (benchmark fractions: *a*/2, *a*/3, or *a*/4, where *a* is a whole number greater than 0 and less than or equal to the denominator) as a part to whole relationship in area and set models where the denominator is equal to the number of parts in the whole **using models, explanations, or other representations**.	**positive fractional numbers** (benchmark fractions: *a*/2, *a*/3, *a*/4, *a*/6, or *a*/8, where *a* is a whole number greater than 0 and less than or equal to the denominator) as a part to whole relationship in area and set models where the number of parts in the whole is equal to the denominator; and **decimals** (within a context of money) as a part of 100 **using models, explanations, or other representations**.	**positive fractional numbers** (benchmark fractions: *a*/2, *a*/3, *a*/4, *a*/5, *a*/6, *a*/8, or *a*/10, where *a* is a whole number greater than 0 and less than or equal to the denominator) as a part to whole relationship in area, set, or linear models where the number of parts in the whole are equal to, and a multiple or factor of the denominator; and **decimals** as hundredths within the context of money, or tenths within the context of metric measurements (e.g., 2.3 cm) **using models, explanations, or other representations**.
M(N&O)–2–2 **Demonstrates understanding of the relative magnitude of numbers** from 0 to 199 by ordering whole numbers; by comparing whole numbers to each other or to benchmark whole numbers (10, 25, 50, 75, 100, 125, 150, or 175); by demonstrating an understanding of the relation of inequality when comparing whole numbers by using “1 more”, “1 less”, “10 more”, “10 less”, “100 more”, or “100 less”; or by connecting number words and numerals to the quantities they represent using models, number lines, or explanations.	M(N&O)–3–2 **Demonstrates understanding of the relative magnitude of numbers** from 0 to 999 by ordering whole numbers; by comparing whole numbers to benchmark whole numbers (100, 250, 500, or 750); or by comparing whole numbers to each other; and comparing or identifying equivalent positive fractional numbers (*a*/2, *a*/3, *a*/4 where *a* is a whole number greater than 0 and less than or equal to the denominator) using models, number lines, or explanations.	M(N&O)–4–2 **Demonstrates understanding of the relative magnitude of numbers** from 0 to 999,999 by ordering or comparing whole numbers; and ordering, comparing, or identifying equivalent proper positive fractional numbers; or decimals using models, number lines, or explanations.
M(N&O)–2–3 **Demonstrates conceptual understanding of mathematical operations involving** addition and subtraction of whole numbers by solving problems involving joining actions, separating actions, part-part whole relationships, and comparison situations; and addition of multiple one-digit whole numbers. (See Appendix A.)	M(N&O)–3–3 **Demonstrates conceptual understanding of mathematical operations** by describing or illustrating the inverse relationship between addition and subtraction of whole numbers; and the relationship between repeated addition and multiplication using models, number lines, or explanations.	M(N&O)–4–3 **Demonstrates conceptual understanding of mathematical operations** by describing or illustrating the relationship between repeated subtraction and division (no remainders); the inverse relationship between multiplication and division of whole numbers; or the addition or subtraction of positive fractional numbers with like denominators using models, number lines, or explanations.

	M(N&O)–3–4 **Accurately solves problems involving** addition and subtraction with and without regrouping; the concept of multiplication; and addition or subtraction of decimals (in the context of money).	M(N&O)–4–4 **Accurately solves problems involving** multiple operations on whole numbers or the use of the properties of factors and multiples; and addition or subtraction of decimals and positive proper fractions with like denominators. (Multiplication limited to 2 digits by 2 digits, and division limited to 1 digit divisors.) (IMPORTANT*: Applies the conventions of order of operations where the left to right computations are modified only by the use of parentheses*.)
M(N&O)–2–5 **Demonstrates understanding of monetary value** by adding coins together to a value no greater than $1.99 and representing the result in dollar notation; making change from $1.00 or less, or recognizing equivalent coin representations of the same value (values up to $1.99).		

Number and Operations

Grade 5	Grade 6	Grade 7	Grade 8 (local curriculum and assessment option)
M(N&O)–5–1 **Demonstrates conceptual understanding of rational numbers with respect to:** **whole numbers** from 0 to 9,999,999 through equivalency, composition, decomposition, or place value **using models, explanations, or other representations**; and **positive fractional numbers** (proper, mixed number, and improper) (halves, fourths, eighths, thirds, sixths, twelfths, fifths, or powers of ten (10, 100, 1000)), **decimals** (to thousandths), or **benchmark percents** (10%, 25%, 50%, 75% or 100%) as a part to whole relationship in area, set, or linear models **using models, explanations, or other representations***.	M(N&O)–6–1 **Demonstrates conceptual understanding of rational numbers with respect to** ratios (comparison of two whole numbers by division a/b, $a : b$, and $a \div b$, where $b \neq 0$); and rates (e.g., a out of b, 25%) **using models, explanations, or other representations***.	M(N&O)–7–1 **Demonstrates conceptual understanding of rational numbers with respect to percents** as a means of comparing the same or different parts of the whole when the wholes vary in magnitude (e.g., 8 girls in a classroom of 16 students compared to 8 girls in a classroom of 20 students, or 20% of 400 compared to 50% of 100); and percents as a way of expressing multiples of a number (e.g., 200% of 50) **using models, explanations, or other representations***.	M(N&O)–8–1 **Demonstrates conceptual understanding of rational numbers with respect to percents** as a way of describing change (percent increase and decrease) **using explanations, models, or other representations***.
***Specifications for area, set, and linear models for grades 5 – 8**: **Fractions:** The number of parts in the whole are equal to the denominator, a multiple of the denominator, or a factor of the denominator. **Percents:** The number of parts in the whole is equal to 100, a multiple of 100, or a factor of 100 (for grade 5); the number of parts in the whole is a multiple or a factor of the numeric value representing the whole (for grades 6-8). **Decimals (including powers of ten)**: The number of parts in the whole is equal to the denominator of the fractional equivalent of the decimal, a multiple of the denominator of the fractional equivalent of the decimal, or a factor of the denominator of the fractional equivalent of the decimal.			
M(N&O)–5–2 **Demonstrates understanding of the relative magnitude of numbers** by ordering, comparing, or identifying equivalent positive fractional numbers, decimals, or benchmark percents within number formats (fractions to fractions, decimals to decimals, or percents to percents); or integers in context using models or number lines.	M(N&O)–6–2 **Demonstrates understanding of the relative magnitude of numbers** by ordering or comparing numbers with whole number bases and whole number exponents (e.g., 3^3, 4^3), integers, or rational numbers within and across number formats (fractions, decimals, or whole number percents from 1- 100) using number lines or equality and inequality symbols.	M(N&O)–7–2 **Demonstrates understanding of the relative magnitude of numbers** by ordering, comparing, or identifying equivalent rational numbers across number formats, numbers with whole number bases and whole number exponents (e.g., 3^3, 4^3), integers, absolute values, or numbers represented in scientific notation using number lines or equality and inequality symbols.	M(N&O)–8–2 **Demonstrates understanding of the relative magnitude of numbers** by ordering or comparing rational numbers, common irrational numbers (e.g., $\sqrt{2}$, π), numbers with whole number or fractional bases and whole number exponents, square roots, absolute values, integers, or numbers represented in scientific notation using number lines or equality and inequality symbols.
M(N&O)–5–3 **Demonstrates conceptual understanding of mathematical operations** by describing or illustrating the meaning of a remainder with respect to division of whole numbers using models, explanations, or solving problems.	M(N&O)–6–3 **Demonstrates conceptual understanding of mathematical operations** by describing or illustrating the meaning of a power by representing the relationship between the base (whole number) and the exponent (whole number) (e.g., 3^3, 4^3); and the effect on the magnitude of a whole number when multiplying or dividing it by a whole number, decimal, or fraction.		
M(N&O)–5–4 **Accurately solves problems involving** multiple operations on whole numbers or the use of the properties of factors, multiples, prime, or composite numbers; and addition or subtraction of fractions (proper) and decimals to the hundredths place. (Division of whole numbers by up to a two-digit divisor.) (IMPORTANT*: Applies the conventions of order of operations with and without parentheses.*)	M(N&O)–6–4 **Accurately solves problems involving** single or multiple operations on fractions (proper, improper, and mixed), or decimals; and addition or subtraction of integers; percent of a whole; or problems involving greatest common factor or least common multiple. (IMPORTANT*: Applies the conventions of order of operations with and without parentheses.*)	M(N&O)–7–4 **Accurately solves problems involving** proportional reasoning; percents involving discounts, tax, or tips; and rates. (IMPORTANT: *Applies the conventions of order of operations including parentheses, brackets, or exponents.*)	M(N&O)–8–4 **Accurately solves problems involving** proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots. (IMPORTANT: *Applies the conventions of order of operations.*)

Geometry and Measurement

Grade 2	Grade 3	Grade 4
M(G&M)–2–1 **Uses properties, attributes, composition, or decomposition to sort or classify** polygons or objects by a combination of two or more non-measurable or measurable attributes.	M(G&M)–3–1 **Uses properties or attributes of angles** (number of angles) **or sides** (number of sides or length of sides) or composition or decomposition of shapes **to identify, describe, or distinguish among** triangles, squares, rectangles, rhombi, trapezoids, hexagons, or circles.	M(G&M)–4–1 **Uses properties or attributes of angles** (number of angles) **or sides** (number of sides, length of sides, parallelism, or perpendicularity) **to identify, describe, or distinguish among** triangles, squares, rectangles, rhombi, trapezoids, hexagons, or octagons; or classify angles relative to 90^o as more than, less than, or equal to.
		M(G&M)–4–3 **Uses properties or attributes** (shape of bases or number of lateral faces) **to identify, compare, or describe three-dimensional shapes** (rectangular prisms, triangular prisms, cylinders, or spheres).
		M(G&M)–4–4 **Demonstrates conceptual understanding of congruency** by matching congruent figures using reflections, translations, or rotations (flips, slides, or turns), or as the result of composing or decomposing shapes using models or explanations.
		M(G&M)–4–5 **Demonstrates conceptual understanding of similarity** by applying scales on maps, or applying characteristics of similar figures (same shape but not necessarily the same size) to identify similar figures, or to solve problems involving similar figures. Describes relationships using models or[sc] explanations.
M(G&M)–2–6 **Demonstrates conceptual understanding of perimeter and area** by using models or manipulatives to surround and cover polygons.	M(G&M)–3–6 **Demonstrates conceptual understanding of perimeter of** polygons, and **the area of** rectangles on grids using a variety of models or manipulatives. Expresses all measures using appropriate units.	M(G&M)–4–6 **Demonstrates conceptual understanding of perimeter of** polygons, and **the area of** rectangles, polygons or irregular shapes on grids using a variety of models, manipulatives, or formulas. Expresses all measures using appropriate units.
M(G&M)–2–7 **Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems** across the content strands. Benchmarks in Appendix B.	M(G&M)–3–7 **Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems** across the content strands. Benchmarks in Appendix B.	M(G&M)–4–7 **Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems** across the content strands. Benchmarks in Appendix B.

Geometry and Measurement			
Grade 5	**Grade 6**	**Grade 7**	Grade 8 **(local curriculum and assessment option)**
M(G&M)–5–1 **Uses properties or attributes of angles** (right, acute, or obtuse) **or sides** (number of congruent sides, parallelism, or perpendicularity) **to identify, describe, classify, or distinguish among** different types of triangles (right, acute, obtuse, equiangular, or equilateral) or quadrilaterals (rectangles, squares, rhombi, trapezoids, or parallelograms).	M(G&M)–6–1 **Uses properties or attributes of angles** (right, acute, or obtuse) **or sides** (number of congruent sides, parallelism, or perpendicularity) **to identify, describe, classify, or distinguish** among different types of triangles (right, acute, obtuse, equiangular, scalene, isosceles, or equilateral) or quadrilaterals (rectangles, squares, rhombi, trapezoids, or parallelograms).	M(G&M)–7–1 **Uses properties of angle** relationships resulting from two or three intersecting lines (adjacent angles, vertical angles, straight angles, or angle relationships formed by two non-parallel lines cut by a transversal), or two parallel lines cut by a transversal to solve problems.	
		M(G&M)–7–2 **Applies theorems or relationships** (triangle inequality or sum of the measures of interior angles of regular polygons) to solve problems.	M(G&M)–8–2 **Applies the Pythagorean Theorem** to find a missing side of a right triangle, or in problem solving situations.
M(G&M)–5–3 **Uses properties or attributes** (shape of bases, number of lateral faces, or number of bases) **to identify, compare, or describe three-dimensional shapes** (rectangular prisms, triangular prisms, cylinders, spheres, pyramids, or cones).	M(G&M)–6–3 **Uses properties or attributes** (shape of bases, number of lateral faces, number of bases, number of edges, or number of vertices) **to identify, compare, or describe three-dimensional shapes** (rectangular prisms, triangular prisms, cylinders, spheres, pyramids, or cones).		
		M(G&M)–7–4 **Applies the concepts of congruency** by solving problems on a coordinate plane involving reflections, translations, or rotations.	
	M(G&M)–6–5 **Demonstrates conceptual understanding of similarity** by describing the proportional effect on the linear dimensions of polygons or circles when scaling up or down while preserving the angles of polygons, or by solving related problems (including applying scales on maps). Describes effects using models or[sc] explanations.	M(G&M)–7–5 **Applies concepts of similarity** by solving problems involving scaling up or down and their impact on angle measures, linear dimensions and areas of polygons, and circles when the linear dimensions are multiplied by a constant factor. Describes effects using models or[sc] explanations.	M(G&M)–8–5 **Applies concepts of similarity** to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate.
M(G&M)–5–6 **Demonstrates conceptual understanding of perimeter** of polygons, **and the area of** rectangles or right triangles through models, manipulatives, or formulas, the area of polygons or irregular figures on grids, **and volume of** rectangular prisms (cubes) using a variety of models, manipulatives, or formulas. Expresses all measures using appropriate units.	M(G&M)–6–6 **Demonstrates conceptual understanding of perimeter** of polygons, **the area of** quadrilaterals or triangles, and **the volume of** rectangular prisms by using models, formulas, or by solving problems; and **demonstrates understanding of the relationships of circle measures** (radius to diameter and diameter to circumference) by solving related problems. Expresses all measures using appropriate units.	M(G&M)–7–6 **Demonstrates conceptual understanding of the area** of circles or the area or perimeter of composite figures (quadrilaterals, triangles, or parts of circles), **and the surface area of** rectangular prisms, **or volume of** rectangular prisms, triangular prisms, or cylinders using models, formulas, or by solving related problems. Expresses all measures using appropriate units.	M(G&M)–8–6 **Demonstrates conceptual understanding of surface area or volume by solving problems involving surface area and volume** of rectangular prisms, triangular prisms, cylinders, or pyramids. Expresses all measures using appropriate units.
M(G&M)–5–7 **Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems** across the content strands. Benchmarks in Appendix B.	M(G&M)–6–7 **Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems** across the content strands. Benchmarks in Appendix B.		

Functions and Algebra

Grade 2	Grade 3	Grade 4	Grade 5
M(F&A)–2–1 **Identifies and extends to specific cases a variety of patterns** (linear and non-numeric) represented in models, tables, or sequences by extending the pattern to the next element, or finding a missing element (e.g., 2, 4, 6, ___, 10).	M(F&A)–3–1 **Identifies and extends to specific cases a variety of patterns** (linear and non-numeric) represented in models, tables, or sequences by extending the pattern to the next one, two, or three elements, or finding missing elements.	M(F&A)–4–1 **Identifies and extends to specific cases a variety of patterns** (linear and nonlinear) represented in models, tables or sequences; and writes a rule in words or[sc] symbols to find the next case.	M(F&A)–5–1 **Identifies and extends to specific cases a variety of patterns** (linear and nonlinear) represented in models, tables, sequences, or in problem situations; and writes a rule in words or[sc] symbols for finding specific cases of a linear relationship.
		M(F&A)–4–3 **Demonstrates conceptual understanding of algebraic expressions** by using letters or symbols to represent unknown quantities to write simple linear algebraic expressions involving any one of the four operations; or by evaluating simple linear algebraic expressions using whole numbers.	M(F&A)–5–3 **Demonstrates conceptual understanding of algebraic expressions** by using letters to represent unknown quantities to write linear algebraic expressions involving any two of the four operations; or by evaluating linear algebraic expressions using whole numbers.
M(F&A)–2–4 **Demonstrates conceptual understanding of equality** by finding the value that will make an open sentence true (e.g., $2 + \square = 7$). (limited to one operation and limited to use addition or subtraction)	M(F&A)–3–4 **Demonstrates conceptual understanding of equality** by showing equivalence between two expressions using models or different representations of the expressions; or by finding the value that will make an open sentence true (e.g., $2 + \square = 7$). (limited to one operation and limited to use addition, subtraction, or multiplication)	M(F&A)–4–4 **Demonstrates conceptual understanding of equality** by showing equivalence between two expressions using models or different representations of the expressions, by simplifying numerical expressions where left to right computations may be modified only by the use of parentheses [e.g., 14 – (2 × 5)] (expressions consistent with the parameters of M(F&A)–4–3), and by solving one-step linear equations of the form $ax = c$, $x \pm b = c$, where a, b, and c are whole numbers with $a \neq 0$.	M(F&A)–5–4 **Demonstrates conceptual understanding of equality** by showing equivalence between two expressions using models or different representations of the expressions (expressions consistent with the parameters of M(F&A)–5–3), by solving one-step linear equations of the form $ax = c$, $x \pm b = c$, or $x/a = c$, where a, b, and c are whole numbers with $a \neq 0$; or by determining which values of a replacement set make the equation (multi-step of the form $ax \pm b = c$ where a, b, and c are whole numbers with $a \neq 0$) a true statement (e.g., $2x + 3 = 11$, $\{x: x = 2, 3, 4, 5\}$).

Functions and Algebra		
Grade 6	**Grade 7**	Grade 8 **(local curriculum and assessment option)**
M(F&A)–6–1 **Identifies and extends to specific cases a variety of patterns** (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; or writes a rule in words or symbols for finding specific cases of a linear relationship; or writes a rule in words or[sc] symbols for finding specific cases of a nonlinear relationship; and writes an expression or[sc] equation using words or[sc] symbols to express the **generalization** of a linear relationship (e.g., twice the term number plus 1 or[sc] $2n + 1$).	M(F&A)–7–1 **Identifies and extends to specific cases a variety of patterns** (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; **and generalizes** a linear relationship using words and symbols; generalizes a linear relationship to find a specific case; or writes an expression or[sc] equation using words or[sc] symbols to express the **generalization** of a nonlinear relationship.	M(F&A)–8–1 **Identifies and extends to specific cases a variety of patterns** (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; **and generalizes** a linear relationship (non-recursive explicit equation); generalizes a linear relationship to find a specific case; **generalizes** a nonlinear relationship using words or[sc] symbols; or **generalizes** a common nonlinear relationship to find a specific case.
M(F&A)–6–2 **Demonstrates conceptual understanding of linear relationships** ($y = kx$; $y = mx + b$) **as a constant rate of change** by constructing or interpreting graphs of real occurrences and describing the slope of linear relationships (faster, slower, greater, or smaller) in a variety of problem situations; **and describes how change in the value of one variable relates to change in the value of a second variable** in problem situations with constant rates of change.	M(F&A)–7–2 **Demonstrates conceptual understanding of linear relationships** ($y = kx$; $y = mx + b$) **as a constant rate of change** by solving problems involving the relationship between slope and rate of change, by describing the meaning of slope in concrete situations, or informally determining the slope of a line from a table or graph; **and distinguishes between constant and varying rates** of change in concrete situations represented in tables or graphs; **or describes how change in the value of one variable relates to change in the value of a second variable** in problem situations with constant rates of change.	M(F&A)–8–2 **Demonstrates conceptual understanding of linear relationships** ($y = kx$; $y = mx + b$) **as a constant rate of change** by solving problems involving the relationship between slope and rate of change; informally and formally determining slopes and intercepts represented in graphs, tables, or problem situations; or describing the meaning of slope and intercept in context; **and distinguishes between linear relationships (constant rates of change) and nonlinear relationships (varying rates of change)** represented in tables, graphs, equations, or problem situations; or **describes how change in the value of one variable relates to change in the value of a second variable** in problem situations with constant and varying rates of change.
M(F&A)–6–3 **Demonstrates conceptual understanding of algebraic expressions** by using letters to represent unknown quantities to write linear algebraic expressions involving two or more of the four operations; or by evaluating linear algebraic expressions (including those with more than one variable); or by evaluating an expression within an equation (e.g., determine the value of y when $x = 4$ given $y = 3x – 2$).	M(F&A)–7–3 **Demonstrates conceptual understanding of algebraic expressions** by using letters to represent unknown quantities to write algebraic expressions (including those with whole number exponents or more than one variable); or by evaluating algebraic expressions (including those with whole number exponents or more than one variable); or by evaluating an expression within an equation (e.g., determine the value of y when $x = 4$ given $y = 5x^3 – 2$).	M(F&A)–8–3 **Demonstrates conceptual understanding of algebraic expressions** by evaluating and simplifying algebraic expressions (including those with square roots, whole number exponents, or rational numbers); or by evaluating an expression within an equation (e.g., determine the value of y when $x = 4$ given $y = 7\sqrt{x} + 2x$).
M(F&A)–6–4 **Demonstrates conceptual understanding of equality** by showing equivalence between two expressions using models or different representations of the expressions (expressions consistent with the parameters of M(F&A)–6–3), solving multi-step linear equations of the form $ax \pm b = c$, where a, b, and c are whole numbers with $a \neq 0$.	M(F&A)–7–4 **Demonstrates conceptual understanding of equality** by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions, solving multi-step linear equations of the form $ax \pm b = c$ with $a \neq 0$, $ax \pm b = cx \pm d$ with $a, c \neq 0$, and $(x/a) \pm b = c$ with $a \neq 0$, where a, b, c and d are whole numbers; or by translating a problem-solving situation into an equation consistent with the parameters of the type of equations being solved for this grade level.	M(F&A)–8–4 **Demonstrates conceptual understanding of equality** by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions, solving formulas for a variable requiring one transformation (e.g., $d = rt$; $d/r = t$); by solving multi-step linear equations with integer coefficients; by showing that two expressions are or are not equivalent by applying commutative, associative, or distributive properties, order of operations, or substitution; and by informally solving problems involving systems of linear equations in a context.

Grade 2	Grade 3	Grade 4
M(DSP)–2–1 **Interprets a given representation** (pictographs with one-to-one correspondence, line plots, tally charts, or tables) to answer questions related to the data, or to analyze the data to formulate conclusions. (IMPORTANT: *Analyzes data consistent with concepts and skills in M(DSP)–2–2.*)	M(DSP)–3–1 **Interprets a given representation** (line plots, tally charts, tables, or bar graphs) to answer questions related to the data, to analyze the data to formulate conclusions, or to make predictions. (IMPORTANT: *Analyzes data consistent with concepts and skills in M(DSP)–3–2.*)	M(DSP)–4–1 **Interprets a given representation** (line plots, tables, bar graphs, pictographs, or circle graphs) to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems. (IMPORTANT: *Analyzes data consistent with concepts and skills in M(DSP)–4–2.*)
M(DSP)–2–2 **Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using** more, less, or equal.	M(DSP)–3–2 **Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using** most frequent (mode), least frequent, largest, or smallest.	M(DSP)–4–2 **Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using** measures of central tendency (median or mode), or range.
	M(DSP)–3–3 **Identifies or describes representations or elements of representations that best display a given set of data or situation**, consistent with the representations required in M(DSP)–3–1.	
M(DSP)–2–4 **Uses counting techniques to solve problems** involving combinations using a variety of strategies (e.g., student diagrams, organized lists, tables, tree diagrams, or[sc] others); (e.g., How many ways can you make 50 cents using nickels, dimes, and quarters?)		M(DSP)–4–4 **Uses counting techniques to solve problems** in context involving combinations or simple permutations (e.g., Given a map – Determine the number of paths from point A to point B.) using a variety of strategies (e.g., organized lists, tables, tree diagrams, or[sc] others).
	M(DSP)–3–5 **For a probability event in which the sample space may or may not contain equally likely outcomes, determines** the likelihood of the occurrence of an event (using “more likely”, “less likely”, or “equally likely”).	M(DSP)–4–5 **For a probability event in which the sample space may or may not contain equally likely outcomes, determines** the theoretical probability of an event and expresses the result as part to whole (e.g., two out of five).

Data, Statistics, and Probability

Grade 5	Grade 6	Grade 7	Grade 8 (local curriculum and assessment option)
M(DSP)–5–1 **Interprets a given representation** (tables, bar graphs, circle graphs, or line graphs) to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems. (IMPORTANT: *Analyzes data consistent with concepts and skills in M(DSP)–5–2.*)	M(DSP)–6–1 **Interprets a given representation** (circle graphs, line graphs, or stem-and-leaf plots) to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems. (IMPORTANT: *Analyzes data consistent with concepts and skills in M(DSP)–6–2.*)	M(DSP)–7–1 **Interprets a given representation** (circle graphs, scatter plots that represent discrete linear relationships, or histograms) to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems. (IMPORTANT: *Analyzes data consistent with concepts and skills in M(DSP)–7–2.*)	M(DSP)–8–1 **Interprets a given representation** (line graphs, scatter plots, histograms, or box-and-whisker plots) to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems. (IMPORTANT: *Analyzes data consistent with concepts and skills in M(DSP)–8–2.*)
M(DSP)–5–2 **Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using** measures of central tendency (mean, median, or mode) or range to analyze situations, or to solve problems.	M(DSP)–6–2 **Analyzes patterns, trends or distributions in data in a variety of contexts by determining or using** measures of central tendency (mean, median, or mode) or dispersion (range) to analyze situations, or to solve problems.	M(DSP)–7–2 **Analyzes patterns, trends, or distributions in data in a variety of contexts by solving problems using** measures of central tendency (mean, median, or mode), dispersion (range or variation), or outliers to analyze situations to determine their effect on mean, median, or mode; and evaluates the sample from which the statistics were developed (bias).	M(DSP)–8–2 **Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using** measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, or estimated line of best fit to analyze situations, or to solve problems; and evaluates the sample from which the statistics were developed (bias, random, or non-random).
M(DSP)–5–3 **Identifies or describes representations or elements of representations that best display a given set of data or situation**, consistent with the representations required in M(DSP)–5–1.		M(DSP)–7–3 **Identifies or describes representations or elements of representations that best display a given set of data or situation**, consistent with the representations required in M(DSP)–7–1.	M(DSP)–8–3 **Organizes and displays data using** scatter plots to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems; or identifies representations or elements of representations that best display a given set of data or situation, consistent with the representations required in M(DSP)–8–1. (IMPORTANT: *Analyzes data consistent with concepts and skills in M(DSP)–8–2.*)
	M(DSP)–6–4 **Uses counting techniques to solve problems** in context involving combinations or simple permutations using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or[sc] others).		M(DSP)–8–4 **Uses counting techniques to solve problems** in context involving combinations or permutations using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or[sc] others).
M(DSP)–5–5 **For a probability event in which the sample space may or may not contain equally likely outcomes, determines** the experimental or theoretical probability of an event and expresses the result as a fraction.	M(DSP)–6–5 **For a probability event in which the sample space may or may not contain equally likely outcomes, determines** the experimental or theoretical probability of an event in a problem-solving situation.	M(DSP)–7–5 **For a probability event in which the sample space may or may not contain equally likely outcomes, determines** the experimental or theoretical probability of an event in a problem-solving situation.	

Appendix A: M(N&O)–2–3

M(N&O)–2–3 Demonstrates conceptual understanding of mathematical operations involving addition and subtraction of whole numbers by solving problems involving joining actions, separating actions, part-part whole relationships, and comparison situations. (See Appendix A.)

Classification of word problems: Researchers have identified four basic categories of addition and subtraction problems: problems with joining actions, separating actions, part-part whole relationships, and comparison situations. Table 1 contains some examples to illustrate the distinctions in the categories identified by the researchers, but there may be additional ways to express the same actions, relationships, or situations. For example, the following are additional ways to ask questions related to the "Join – Start Unknown" category.

> **Example 1:** "I am thinking of a number that when you add 5 to it, the answer is 13. What number am I thinking of?"
> **Example 2:** John puts a five-pound toy inside an empty wooden box. The box and toy together weigh 7 pounds. How much does the empty box weigh?

In summary, students should have experiences with problem solving in addition and subtraction across a variety of problem types. It is important to remember that any problem situation that fits the equations given and the category can be asked, not just problems as stated in Table 1.

"Teaching students to add and subtract involves providing students with an opportunity to explore a rich set of problems with different semantic structures and to develop a variety of strategies to quantify, represent, calculate, express, and justify results." (Gutstein, E., Romberg, 1995)

Table 1: Classification of Word Problems[1]

<table>
<tr><td>Join</td><td>(Result Unknown)
Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether?

5 + 8 =</td><td colspan="2">(Change unknown)
Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether?

5 + = 13</td><td>(Start Unknown)
Connie has some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?

+ 5 = 13</td></tr>
<tr><td>Separate</td><td>(Result Unknown)
Connie had 13 marbles. She gave 5 marbles to Juan. How many marbles does she have left?

13 – 5 =</td><td colspan="2">(Change unknown)
Connie has 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give Juan?

13 – = 5</td><td>(Start Unknown)
Connie has some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with?

– 5 = 8</td></tr>
<tr><td>Part-Part-Whole</td><td colspan="2">(Whole Unknown)
Connie has 5 red marbles and 8 blue marbles. How many marbles does Connie have?

5 + 8 =</td><td colspan="2">(Part Unknown)
Connie has 13 marbles. All the marbles are either blue or red. Connie has 5 red marbles. How many blue marbles does Connie have?

13 – 5 =</td></tr>
<tr><td>Compare</td><td>(Difference Unknown)
Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan?

13 – 5 =</td><td colspan="2">(Compare Quantity Unknown)
Juan has 5 marbles. Connie has 8 more marbles than Juan. How many marbles does Connie have?

5 + 8 =</td><td>(Referent Unknown)
Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have?

13 – 5 =</td></tr>
</table>

[1] Carpenter, Fennema, Peterson, Chiang, and Loef (1989) cited in Gutstein, E., Romberg, T., *Teaching Children to Add and Subtract*, Journal of Mathematical Behavior, 14, 283-324 (1995).

Appendix B: Measurement Benchmarks

The following is a list of the measurement benchmarks and equivalences that ***can be used*** in problems across the content strands at each grade level to address the expectations in M(G&M)–X–7 for the NECAP Assessment[2].

M(G&M)–X–7 **Uses units of measures appropriately and consistently, and makes conversions within systems when solving problems** across the content strands.

The type of measure (e.g., length, time, etc.), the unit (e.g., inches, feet, etc.), the degree of accuracy where appropriate (e.g., ½ inch); and equivalences (e.g., 12 inches in a foot) are identified for grades 2 – 8. In addition to measurement benchmarks identified below students will be expected to use the appropriate units when solving problems involving area, volume, surface area, conversions, and rates (e.g., miles per hour, price per pound, pounds per square inch) on the NECAP Assessment[1].

Measures	**Grade 2**	**Grade 3**	**Grade 4**
Length	**Unit (accuracy):** Inch (to whole inch); Foot (to whole inch); Centimeter (to whole centimeter); Meter (to whole centimeter) **Equivalencies:** 12 inches in 1 foot; 100 centimeters in 1 meter	**Unit (accuracy):** Inch (to 1/2 inch); Foot (to whole inch); Centimeter (to whole centimeter); Meter (to whole centimeter) **Equivalencies:** 12 inches in 1 foot; 100 centimeters in 1 meter	**Unit (accuracy):** Inch (to 1/4 inch); Foot; Centimeter (to 0.5 centimeter); Meter (to 0.5 centimeter); Yard; Mile (use in scale questions); Kilometer (use in scale questions) **Equivalencies:** 12 inches in 1 foot; 100 centimeters in 1 meter; 3 feet in 1 yard; 36 inches in 1 yard
Time	**Unit (accuracy):** Hour (to 15 minute interval) **Equivalencies:** 60 minutes in 1 hour	**Unit (accuracy):** Hour (to 5 minute interval); Day; Year **Equivalencies:** 24 hours in 1 day; 7 days in 1 week; 365 days in 1 year	**Unit (accuracy):** Hour (to 5 minute interval); Day; Year **Equivalencies:** 24 hours in 1 day; 7 days in 1 week; 365 days in 1 year; 60 seconds in 1 minute; 60 minutes in 1 hour
Temperature	**Unit (accuracy):** Degree (to 1 degree)	**Unit (accuracy):** Cº and Fº (to 1 degree)	**Unit (accuracy):** Cº and Fº (to 1 degree)
Capacity		**Units (accuracy):** Quart (to whole quart)	**Unit (accuracy):** Quart (to whole quart)
Mass		**Unit (accuracy):** Kilogram (to whole kilogram); Gram (to whole gram)	**Unit (accuracy):** Kilogram (to whole kilogram); Gram (to whole gram)
Weight		**Unit (accuracy):** Pound (to whole pound)	**Unit (accuracy):** Pound (to whole pound)

[2] Contractors will be asked to devise a system to measure the degree to which students use units of measures and make conversions consistently and appropriately when applicable to problems across content strands.

Measures	Grade 5	Grades 6 – 8
Length	**Units (accuracy):** Inch (to 1/8 inch); Foot; Centimeter (to 0.5 centimeter); Meter (to 0.5 centimeter); Yard; Mile (use in scale questions); Kilometer (use in scale questions) **Equivalencies:** 12 inches in 1 foot; 100 centimeters in 1 meter; 3 feet in 1 yard; 36 inches in 1 yard; 10 millimeters in 1 centimeter	**Units (accuracy):** Inch (to 1/16 inch); Foot; Centimeter (to 1/10 centimeter); Meter (to 1/100 meter); Yard; Mile (use in scale and rate questions); Kilometer (use in scale and rate questions) **Equivalencies:** 12 inches in 1 foot; 100 centimeters in 1 meter; 3 feet in 1 yard; 36 inches in 1 yard; 10 millimeters in 1 centimeter; 1000 millimeters in 1 meter
Time	**Unit (accuracy):** Hour (to 1 minute); Day; Year **Equivalencies:** 24 hours in 1 day; 7 days in 1 week; 365 days in 1 year; 60 seconds in 1 minute; 60 minutes in 1 hour	**Unit (accuracy):** Hour (to 1 minute); Day; Year **Equivalencies:** 24 hours in 1 day; 7 days in 1 week; 365 days in 1 year; 60 seconds in 1 minute; 60 minutes in 1 hour
Temperature	**Unit (accuracy):** Cº and Fº (to 1 degree)	**Unit (accuracy):** Cº and Fº (to 1 degree)
Capacity	**Unit (accuracy):** Quart (to 1 ounce); Gallon; Pint **Equivalencies:** 32 ounces in 1 quart; 4 quarts in 1 gallon; 2 pints in 1 quart	**Unit (accuracy):** Quarts (to 1 ounce); Gallon; Pint; Liter **Equivalencies:** 32 ounces in 1 quart; 4 quarts in 1 gallon; 2 pints in 1 quart; 1000 milliliters in 1 liter
Mass	**Unit (accuracy):** Kilogram; Gram (to whole gram)	**Unit (accuracy):** Kilogram; Gram (to 1/10 gram)
Weight	**Unit (accuracy):** Pound (to 1 ounce) **Equivalencies:** 16 ounces in 1 pound	**Unit (accuracy):** Pound (to 1 ounce) **Equivalencies:** 16 ounces in 1 pound
Angles and Rotation	**Unit (accuracy):** Degree (to 2 degrees)	**Unit (accuracy):** Degree (to 2 degrees) **Equivalencies:** 360º in 1 circle; 90º in 1 right angle