Resource Material Prototype

Mathematics GLE Resource Materials
Definitions and Examples for Grades K - 8

Number and Operations

November 2004

New Hampshire Department of Education
Rhode Island Department of Education
Vermont Department of Education

Resource Material Prototype

Overview

The purpose of these resource materials is to provide K – 8 educators with explanations and examples that facilitate understanding of the mathematics in the Grade-Level Expectations (GLEs)*. These resource materials are organized by content strand and by GLE "**stems**." The definitions and explanations within each section are not alphabetized, but are organized to parallel the introduction of new concepts and skills within each GLE "**stem**" across grades. *The materials contained in this document focus on the Number and Operation strand and only address the NECAP GLEs that are common to all three states.*

Stem

Grade 3	Grade 4
M(N&O)–3–3 **Demonstrates conceptual understanding of mathematical operations** by describing or illustrating the inverse relationship between addition and subtraction of whole numbers; and the relationship between repeated addition and multiplication using models, number lines, and explanations.	M(N&O)–4–3 **Demonstrates conceptual understanding of mathematical operations** by describing or illustrating the relationship between repeated subtraction and division (no remainders); the inverse relationship between multiplication and division of whole numbers; or the addition and subtraction of positive fractional numbers with like denominators using models, number lines, or explanations.

To facilitate access, each definition is coded (e.g., ***N&O – 4***, the 4th definition for the Number and Operations strand). There are multiple tables of contents: 1) An overall table of contents on page 3 contains all the terms or phrases defined for the content strand in alphabetical order; and 2) A table of contents for each section in alphabetical order. In addition, if there is a word or phrase that you are unclear about within a definition, check the overall table of contents to determine if the word or phrase is defined in another location in this document.

Section #	Grade-Level Expectation "Stem"	NECAP GLE Stem Codes (Vermont codes) X represents grades K– 8	Page Numbers
Section 1	Demonstrates understanding of rational numbers…	NECAP: M(N&O) – X – 1 (Vermont GE MX: 1)	4 – 18
Section 2	Demonstrates conceptual understanding of the magnitude of numbers…	NECAP: M(N&O) – X – 2 (Vermont GE MX: 2)	19 – 25
Section 3	Demonstrates understandings of mathematical operations…	NECAP: M(N&O) – X – 3 (Vermont GE MX: 3)	26 – 29
Section 4	Accurately solves problems involving…	NECAP: M(N&O) – X – 4 (Vermont GE MX: 4)	30 – 36

Note: Examples are provided throughout this document to illustrate definitions or phrases used in the mathematics GLEs. However, the kinds of questions students might be asked in instruction or on the NECAP assessment about the mathematics being illustrated are NOT necessarily limited to the *specific* examples given. It is the intent, over time, to add released NECAP items to expand the set of examples.

Please send comments, corrections, or suggestion to mathsurvey@gmavt.net.

* Grade-Level Expectations are called Grade Expectations (GEs) in Vermont.

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Section 1: Demonstrates Understanding of Rational Numbers

NECAP: M(N&O) – X – 1
Vermont: MX: 1

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N&0 – 1 **Rational Number:** A rational number is any number that can be represented in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Rational numbers include whole numbers, integers, fractions that when expressed as decimals terminate or repeat, and decimals that are terminating or repeating. (See ***N&O – 11*** for a diagram which illustrates how the set of rational numbers is related to other sets of numbers in the real number system.)

Example 1.1 – Rational numbers:

$$3 = \frac{3}{1} = 3.0;\ \frac{3}{4} = 0.75;\ \frac{1}{2} = 0.5;\ \frac{1}{3} = 0.\bar{3};\ \frac{-2}{7} = -0.\overline{285714};\ -5 = -\frac{5}{1} = 5.0$$

The numeral 3 is a rational number since it can be represented as $\frac{3}{1}$.

N&0 – 2 **Whole number:** A whole number is any number in the set {0, 1, 2, 3, 4, 5, ...}. (See ***N&O – 11*** for a diagram which illustrates how the set of whole numbers is related to other sets of numbers in the real number system.)

N&0 – 3 **Fraction:** A fraction is a quotient of one number or expression to another denoted by $\frac{a}{b}$, when a is the dividend or numerator and b is the divisor or denominator.

A whole number is not typically called a fraction until it is written in fractional form. (e.g., $3 = \frac{3}{1}$)

3 ← The top number of the fraction is the numerator or dividend.

4 ← The bottom number of the fraction is the denominator or divisor.

Note: Not all fractions are rational numbers (e.g., $\frac{\sqrt{3}}{2}$ is a fraction since it is a quotient of one number to another, but is not a rational number since $\sqrt{3}$ is irrational (See ***N&0 – 10***)).

(Definition *N&O – 3* continued on following page)

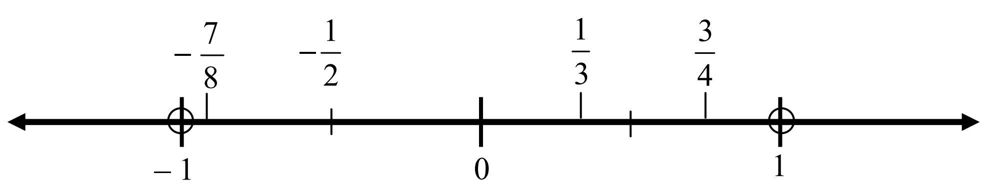
Fraction Notation in GLEs: The notation at grade 2, for example, is as follows: $\frac{a}{2}$, $\frac{a}{3}$, or $\frac{a}{4}$ where a is whole number greater than 0 and less than or equal to the denominator.

Grade 2 Fractions	
$\frac{a}{2}$	$\frac{1}{2}$ and $\frac{2}{2}$
$\frac{a}{3}$	$\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$
$\frac{a}{4}$	$\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$

Fractions can be expressed as proper fractions, improper fractions, and mixed numbers.

N&0 – 4 **Proper Fraction:** A proper fraction is a fraction whose numerator is less in absolute value than its denominator. All proper fractions lie between –1 and 1 on a number line (Note: Zero is considered a proper fraction when written in its fractional form (e.g., $\frac{0}{1}$, $\frac{0}{2}$, $\frac{0}{3}$,...). (*See N&O – 23 for a definition of absolute value.*)

Example 4.1 – Proper fractions:



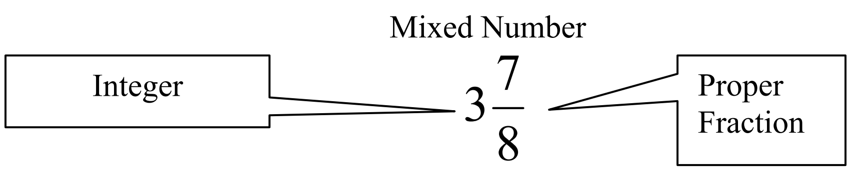
N&0 – 5 **Improper Fraction:** An improper fraction is a fraction whose numerator is greater than or equal to its denominator in absolute value. (*See N&O – 23 for a definition of absolute value.*)

Example 5.1 – Improper fractions:

$$\frac{8}{8};\ -\frac{12}{3};\ \frac{13}{3}$$

N&0 – 6 **Mixed Number:** A mixed number is the sum of an integer and a proper fraction.

Example 6.1: $3\frac{7}{8}$ means $3+\frac{7}{8}$.



N&0 – 7 **Decimal:** A decimal can represent a rational or irrational numbers. A decimal that either terminates or repeats represents a rational number. A decimal that does not terminate or repeat represents an irrational number.

N&0 – 8 **Percent:** Percent is a term meaning per hundred. Percent is denoted with a % symbol.

Example 8.1:

$$7\% = \frac{7}{100} = 0.07$$

N&0 – 9 **Integer:** An integer is any number in the set {…, –3, –2, –1, 0, 1, 2, 3, …}. (See ***N&O – 11*** for a diagram which illustrates how the set of integers is related to other sets of numbers in the real number system.)

N&0 – 10 **Irrational Number:** An irrational number is any real number that is not rational (i.e., Irrational numbers are real numbers whose decimal representations neither terminate nor repeat.). (See ***N&O – 11*** for a diagram which illustrates how the set of irrational numbers is related to other sets of numbers in the real number system.)

Example 10.1 – Irrational numbers:

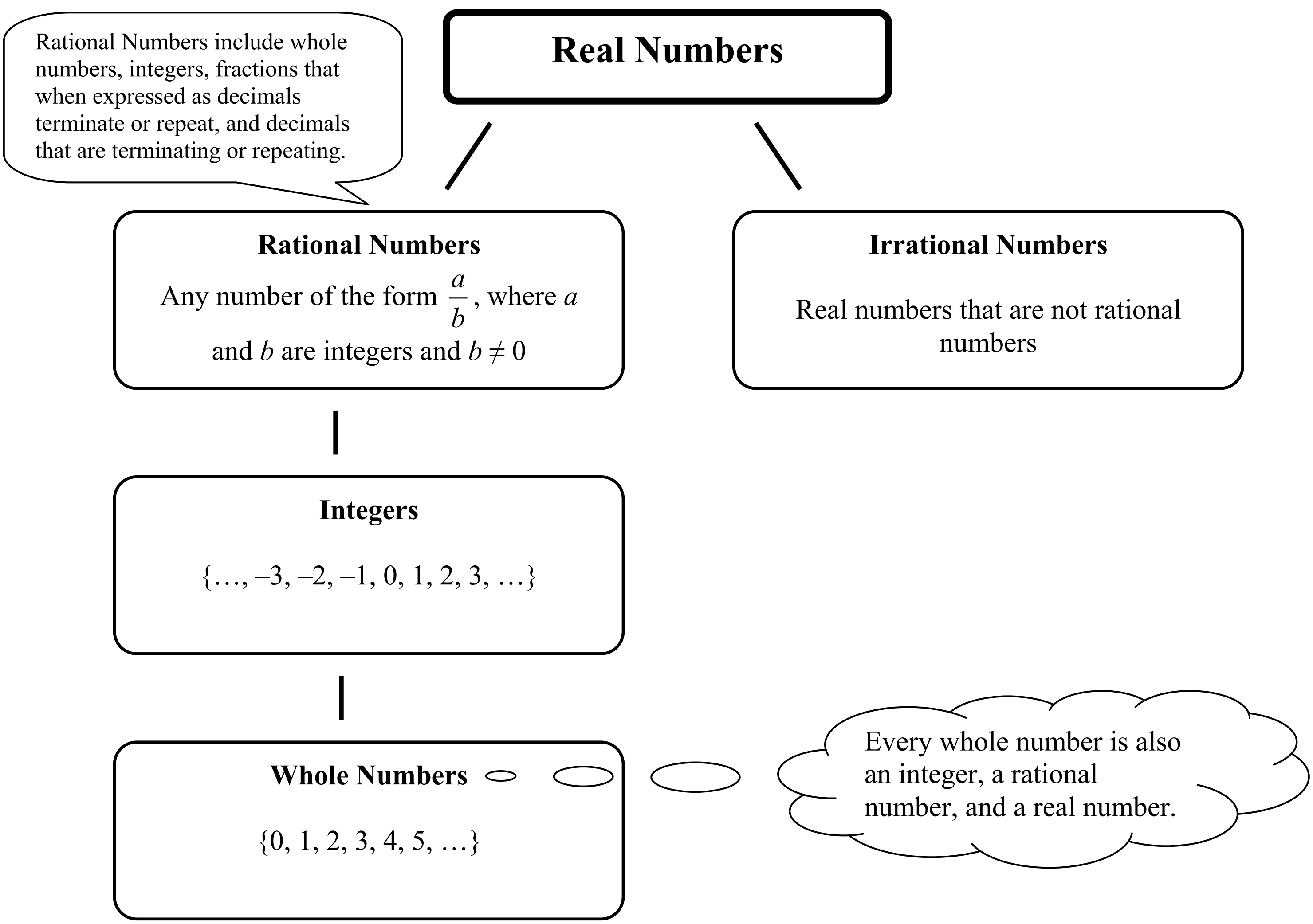
$$\sqrt{2},\ \pi,\ e,\ \frac{\sqrt{2}}{4},\ \sqrt{3}$$

N&0 – 11 **Real Numbers:** Real numbers include rational and irrational numbers.

The following diagram shows how the subsets of the real number system (that have been defined in this section) are related to each other.

The diagram shows, for instance, that every whole number is also an integer, a rational number, and a real number; however, not every integer is a whole number (e.g., –3 is an integer, but not a whole number); not every rational number is an integer (e.g., $\frac{2}{5}$ is a rational number, but not an integer); no irrational number is a rational number; every integer is a rational number (e.g., $2 = \frac{2}{1}$), and so on.

Relationships Between Some of the Subsets of the Real Number System



N&0 – 12 **Ratio:** A ratio is a quotient of two numbers or quantities. Ratios can compare similar units of measures (e.g., blue cars to red cars), or unlike units of measures (e.g., 100 miles per 2 hours). Ratios that compare unlike units are called rates.

The ratio of a to b is typically written as $\frac{a}{b}$ or a:b. Even though we write ratios in the form $\frac{a}{b}$ or a:b to indicate relative size, numbers in other forms can be interpreted as ratios (e.g., 20% represents the ratio $\frac{1}{5}$ or 1:5). Ratios can express part to whole or part to part relationships.

Example 12.1 – Ratios:

There are 28 students in a fifth grade class. Ten students have blue eyes. Fourteen students have brown eyes. Four students have hazel eyes.

Example of part to whole ratio: The ratio of the students in the class with hazel eyes to the students in the whole class is 4:28 or 1:7. The ratio 1:7 means for every one student with hazel eyes there are 7 students in the class.

Example of a part to part ratio: The ratio of the students in the class with blue eyes to the students in the class with brown eyes is 10:14 or 5:7. The ratio 5:7 means for every 5 students in the class with blue eyes there are 7 students in the class with brown eyes.

Example 12.2 – Demonstrates understanding of ratios (part to whole):

Dana and Jamie ran for Student Council President at Midvale Middle School. The data below represent the voting results for grade 7 and grade 8.

	7th Grade Votes		**8th Grade Votes**	
	Jamie	Dana	Jamie	Dana
Boys	24	40	25	42
Girls	49	20	19	40

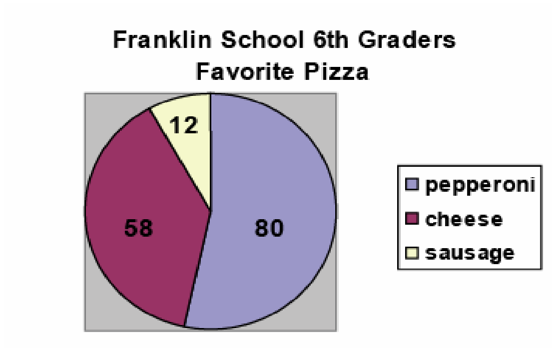
John says that the ratio of the 7th grade boys who voted for Jamie to the seventh grade students who voted for Jamie is about 1:2. Mary disagrees. She says it is about 1:3. Who is correct? Explain your answer.

Answer: Mary is correct. John provided the ratio of boys to girls who voted for Jamie (24 boys:49 girls is about 25:50 or 1:2). Mary provided the ratio of boys who voted for Jamie to all the seventh grade students who voted for Jamie (24 boys:73 seventh grade students is about 25:75 or 1:3).

(Definition *N&O – 12* continued on following page)

Example 12.3 – Demonstrates understanding of ratios (part to part):

The graph below shows the number of 6th grade students at Franklin School and their favorite pizza toppings (each student voted for only one topping).



Which statement about the 6th grade students at Franklin School is true?

A) The ratio of students who prefer sausage pizza to the students who prefer cheese pizza is about 1:3.
B) The ratio of students who prefer sausage pizza to the students who prefer pepperoni pizza is about 1:5.
C) The ratio of students who prefer cheese pizza to the students who prefer pepperoni pizza is about 1:2.
D) The ratio of students who prefer cheese pizza to the students who prefer sausage pizza is about 5:1.

Answer: D; The ratio 58:12 is the same as 29:6, which is about 5:1.

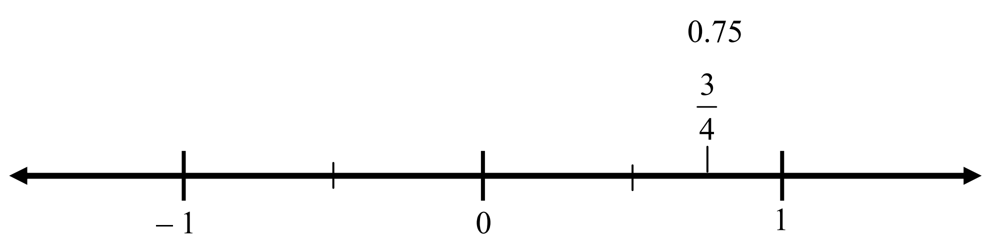
N&0 – 13 **Expanded notation:** In expanded notation a number is represented by the sum of the values of each digit in the number.

Example 13.1:

367 = 300 + 60 + 7	(Grades 1 – 3)
2,367 = $(2 \cdot 1000) + (3 \cdot 100) + (6 \cdot 10) + (7 \cdot 1)$	(Grades 4 – 5)
2,367 = $(2 \cdot 10^3) + (3 \cdot 10^2) + (6 \cdot 10^1) + (7 \cdot 10^0)$	(Grade 6 – 8)

N&0 – 14 **Equivalent numbers:** Two numbers are equivalent if they share the same position on a number line (e.g., $\frac{1}{2} = 0.5 = 50\%$; $\frac{1}{2} = \frac{6}{12}$; 34 = 17 + 17).

Example 14.1: $\frac{3}{4}$ and 0.75 are located at the same place on the number line. Therefore, they are equivalent.



N&0 – 15 **Composition of numbers:** Demonstrating understanding of whole numbers using composition means that a whole number can be composed by adding two or more numbers (e.g., 3 + 5 = 8; 6 + 2 = 8; 2 + 2 + 4 = 8).

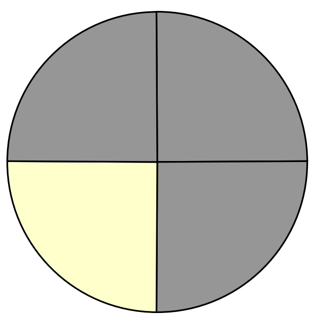
N&0 – 16 **Decomposition of numbers:** Demonstrating understanding of whole numbers using decomposition means that a whole number can be decomposed into multiple addends (e.g., 8 = 3 + 5; 8 = 6 + 2; 8 = 2 + 2 + 4).

Area, Set, and Linear Models

N&0 – 17 through N&O – 19 focus on the specifications in the GLEs for area, set, and linear models. Examples 17.1 through 19.2 represent the intent of the specifications for area, set, and linear models. In spirit, students should have experience with a variety of models (area, set, and linear), and with a variety of relationships (e.g., multiple or factor of the denominator of a fraction) between the number of parts in the model and the fraction, decimal, or percent being represented. In addition, for the most part, each of these examples can be slightly changed to focus on fractions, decimals, or percents. For example, Example 18.2 might be rewritten to "Circle 50% of the butterflies." Also, "part to whole" is used loosely in the GLEs to mean "part to whole", "whole to part", and "part to part" relationships.

N&0 – 17 **Area model to represent part to whole relationships:** An area model can be used to represent part to whole relationships for fractions, decimals, and percents. The entire model may represents the whole, where the model is divided into parts of equal area (e.g., Example 17.3), the model given may represent a part where the whole is to be determined (e.g., If □ represents 3/4, draw a model that represents 1), or the model given may represent a part where another part is to be determined (e.g., If □ represents 3/4, draw a model that represents 2/3). Examples 17.1 – 19.2 focus on clarifying the relationship between the number of parts in the model and the fraction, decimal, or percent being represented.

Example 17.1: What fraction of the circle is shaded gray?



In this example, $\frac{3}{4}$ of the circle is shaded gray. Therefore, the number of parts in the whole, 4, is equal to the denominator.

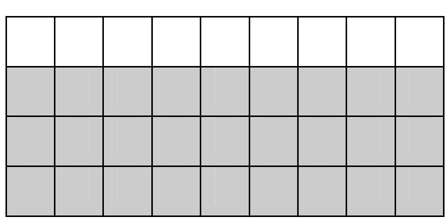
Example 17.2: What fraction of the figure is shaded?

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $\frac{3}{4}$

D. $\frac{1}{4}$



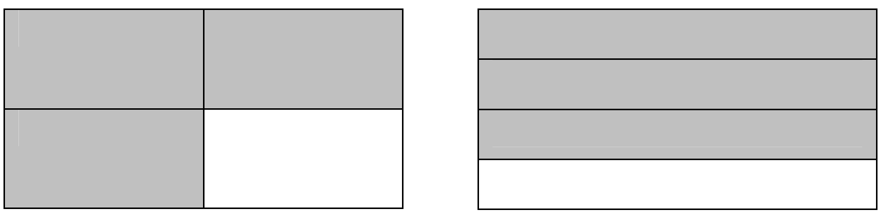
In this example, $\frac{3}{4}$ is the correct answer and the number of parts in the whole, 36, is a multiple of the denominator, 4.

(Definition *N&O – 17* continued on following page)

Example 17.3: Shade $\frac{3}{4}$ of the figure below.

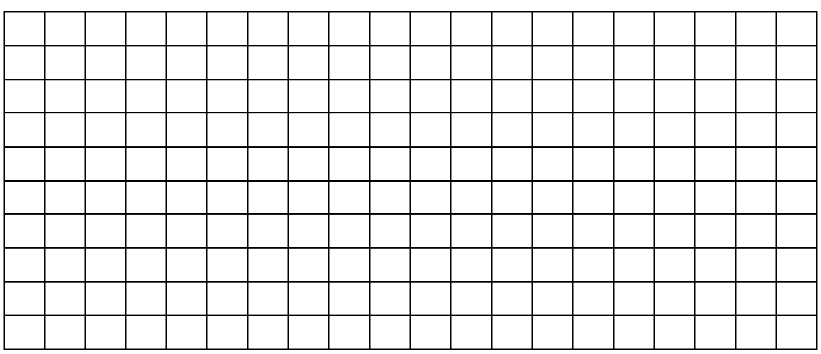
In this example, the number of parts in the whole, 2, is a factor of the denominator, 4.

Answer: These are two ways that students could shade $\frac{3}{4}$ of the figure. Students may also shade $\frac{3}{4}$ of the figure using other strategies.



Example 17.4: Shade 75% of the grid.

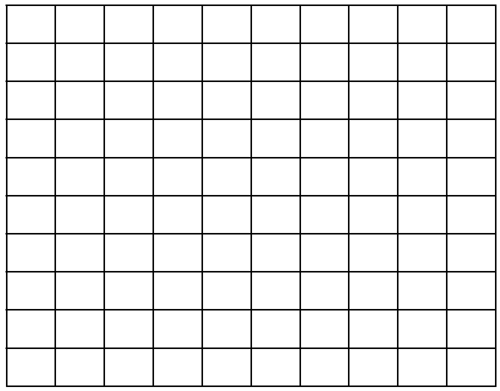
In this example, the number of parts in the whole (the grid), 200, is a multiple of 100.



Answer: Any 150 small squares (□) in the grid should be shaded.

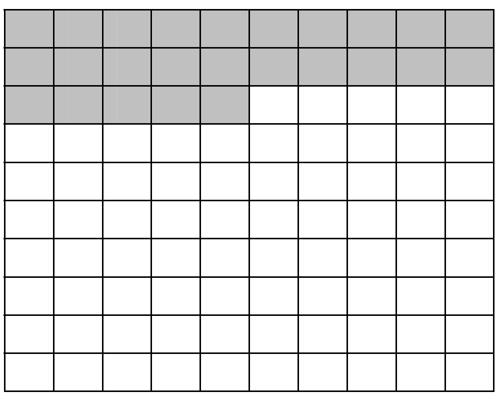
(Definition *N&O – 17* continued on following page)

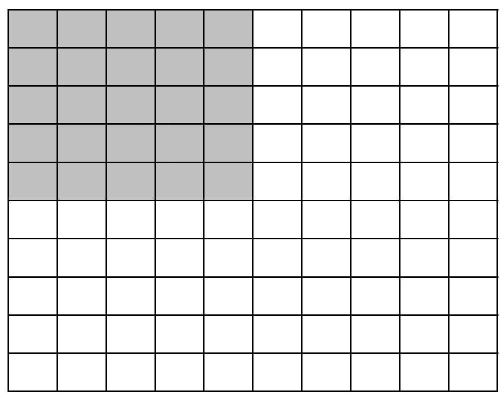
Example 17.5: Shade 25% of the grid.



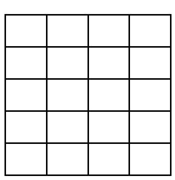
In this example, the number of parts in the whole (the grid) is equal to 100.

Answer: These are two ways that students could shade 25% of the grid. Students may also shade 25% of the grid using other strategies as long as 25 small squares (□) are shaded.





Example 17.6: Shade 25% of the grid.



In this example, the number of parts in the whole (the grid), 20, is a factor of 100.

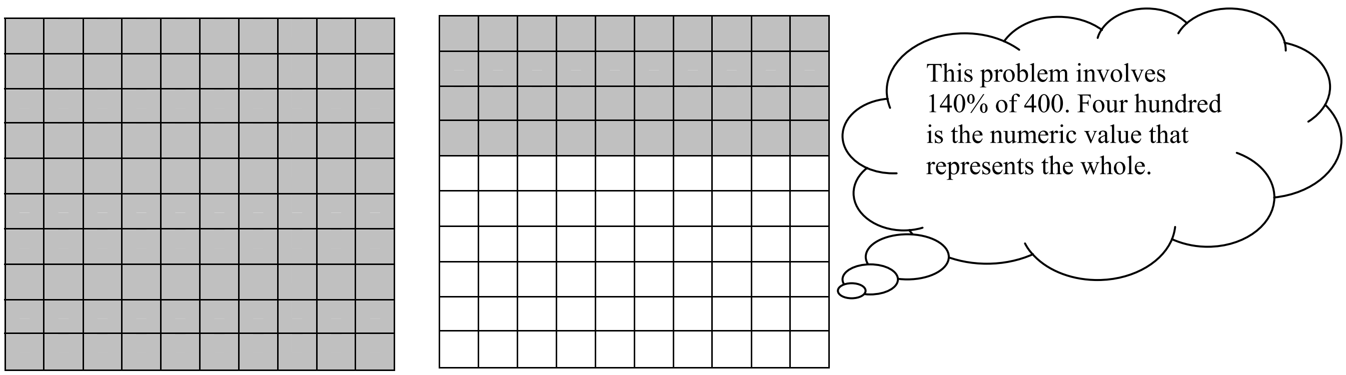
Answer: Any 5 small squares (□) should be shaded.

(Definition *N&O – 17* continued on following page)

Example 17.7:

This year's school enrollment is 560 students. This represents a 40% increase from the last year's school enrollment. The shaded region in the model below represents this year's school enrollment.

What was last year's school enrollment? Explain how to use the model to determine last year's school enrollment.



Answer: 400;

Number of students represented by each box (□) is $\frac{560}{140} = 4$ students

Number of students represented by 100 boxes (last year's population) = 4 students · 100 boxes = 400 students.

Adapted from NCTM Illuminations – A Conceptual Model for Solving Percents (Problem 4)

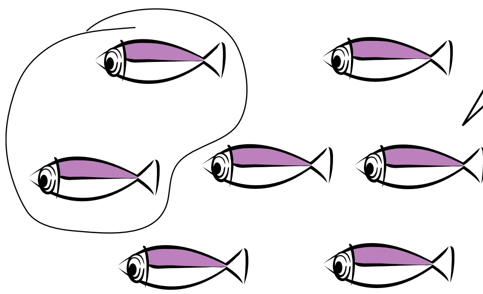
Go to **http://illuminations.nctm.org/index_d.aspx?id=249** for other examples of conceptual percent problems that use models.

N&0 – 18 **Set model:** Since a set is a collection of objects, "demonstrating understanding of part to whole relationship in a set model" means to identify a fractional part of a set, or identify the fraction represented. Additionally, as with area models (see *N&0 – 17*), the set given may represent a part where the whole is to be determined (e.g., if the butterflies in Example 18.2 represent 2/3 of a set of butterflies, then 9 butterflies represent the entire set), or the set given may represent a part where another part is to be determined (e.g., if the butterflies in Example 18.2 represent 2/3 of a set, then 3 butterflies represent 1/3 of the set).

Example 18.1 – Name the fraction of a set that is identified:

What fraction of the set of fish is circled?

In this example, the set of fish represents the whole and the number of parts in the whole, 7, is equal to the denominator.



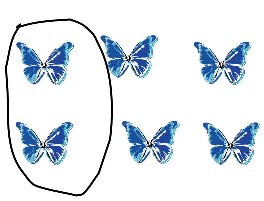
Answer: $\frac{2}{7}$ of the fish are circled.

Example 18.2 – Identify the fractional part of a set:

In this example, the set of butterflies represents the whole and the number of parts in the whole, 6, is a multiple of the denominator, 3.

Circle $\frac{1}{3}$ of the butterflies.

Solution:

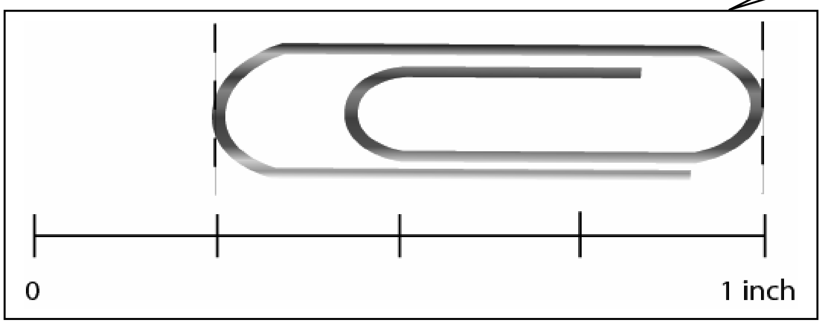


N&0 – 19 **Linear model:** Linear models include number lines, scales (temperature), and linear measurements. Linear models can be used in a similar fashion as area and set models (see *N&0 – 17* and *N&0 – 18*).

Example 19.1:

What is the length of the paper clip below?

In this example, the number of parts in the whole, 4, is equal to the denominator, 4.

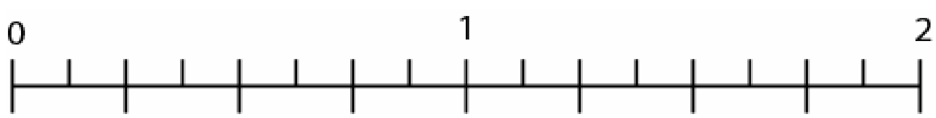


Answer: $\frac{3}{4}$ inch

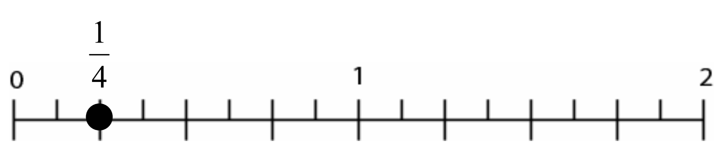
Example 19.2:

Place $\frac{1}{4}$ on the number line below in the correct location.

In this example, the number of parts in ***each*** whole, 8, is a multiple of the denominator, 4.



Answer:



Resource Material Prototype

Section 2: Demonstrates Understanding of Relative Magnitude of Numbers

NECAP: M(N&O) – X – 2
Vermont: MX: 2

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Whole numbers	*	*N&O – 2*
Within number formats	21	*N&O – 21*

*** Found in Number and Operations Section 1**

N&0 – 20 **Relative magnitude:** Relative magnitude is the property of relative size. It refers to the relative size of numbers, objects, distances, brightness, and other things that can be quantified. The GLEs specifically require students to demonstrate understanding of the relative magnitude of different types of numbers by comparing, ordering, and identifying equivalent forms of numbers within and across number formats using models, number lines, equality (=) and inequality symbols (≠, ≤, ≥, >, <), and explanations. (See GLEs for specifics by grade.)

Table 1: Number Formats by Grade for Demonstrating Relative Magnitude
(For grades K-1, consult your state's Local GLEs.)

Grade	Number Formats	Within Number Formats	Across Number Formats
2	○ Whole numbers 0 to 199	√	
3	○ Whole numbers 0 to 999 ○ Positive fractions (halves, thirds, or fourths)	√	
4	○ Whole numbers 0 to 999,999 ○ Positive fractions (halves, thirds, fourths, fifths, sixths, eighths, or tenths) ○ Decimals (to hundredths place)	√	
5	○ Whole numbers 0 to 9,999,999 ○ Positive fractions (halves, fourths, eighths, thirds, fourths, sixths, twelfths, fifths, or powers of 10) ○ Decimals (to thousandths) ○ Benchmark percents (10%, 25%, 50%, 75%, or 100%) ○ Integers in context	√	
6	○ Numbers with whole number bases and whole number exponents (e.g., 3^4 compared to 4^3) ○ Integers ○ Rational numbers (fractions, decimals, whole number percents from 1 – 100%)	√	√
7	○ Numbers with whole number bases and whole number exponents (e.g., 3^4 compared to 4^3) ○ Integers ○ Rational numbers ○ Absolute values ○ Numbers in scientific notation	√	√
8	○ Numbers with whole number of fractional bases and whole number exponents (e.g., 3^4 compared to $(\frac{1}{3})^4$) ○ Integers ○ Rational numbers ○ Absolute values ○ Square roots ○ Numbers in scientific notation ○ Common irrational numbers (e.g., $\sqrt{2}, \pi$)	√	√

N&0 – 21 **Within number formats:** To compare numbers within number formats means to compare whole numbers to whole numbers, fractions to fractions, decimals to decimals, and so on.

N&0 – 22 **Across number formats**: To compare numbers across number formats means to compare whole numbers to fractions, fractions to decimals, decimals to percents, and so on.

Rational numbers: (See ***N&0 – 1***.)

Whole Numbers: (See ***N&0 – 2***.)

Positive fractions: (See ***N&0 – 3, N&0 – 4, N&0 – 5, N&0 – 6***.)

Decimals: (See ***N&0 – 7***.)

Percents: (See ***N&0 – 8***.)

Integers: (See ***N&0 – 9***.)

Real Numbers: (See ***N&0 – 11***.)

Irrational Numbers: (See ***N&0 – 10***.)

N&0 – 23 **Absolute value:** The absolute value of a real number is the distance between 0 and the number on the number line. The absolute value of a real number x is written as $|x|$. The absolute value of a non-negative number (a number greater than or equal to zero) is the number (e.g., $|5| = 5$ since the number 5 is a distance of five units from on the number line). The absolute value of a negative number is the opposite of the number (e.g., $|-5| = 5$ since the number – 5 is five units from 0 on the number line).

N&0 – 24 **Whole number bases and whole number exponents, and fractional bases with whole number exponents:** A whole number exponent is a whole number (See *N&O – 2*) that indicates repeated multiplication of the same number. The number being multiplied is called the base. The exponent is typically written to the right of the base and slightly raised. The exponent indicates how many times the base is used as a factor (See *N&O – 38)*.

Example 24.1 – Whole number base and whole number exponent:

The whole number exponent is 3.

$$6^3 = \underbrace{6 \times 6 \times 6}_{\text{3 factors}} = 216$$

The whole number base is 6.

Example 24.2 – Fractional base with whole number exponent:

The whole number exponent is 4.

$$\left(-\frac{1}{5}\right)^4 = \underbrace{\left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{5}\right)}_{\text{4 factors}} = \frac{1}{625}$$

The fractional base is $-\frac{1}{5}$.

N&0 – 25 **Scientific notation:** Scientific notation is a way of representing very large or very small numbers as the product of a number, *n*, and a power of 10, where the absolute value of *n* is greater than or equal to 1 and less than 10.

Examples 25.1:

The absolute value of –1.21 is greater than or equal to 1 and less than 10.

10^{-4} means $\frac{1}{10^4} = \frac{1}{10{,}000}$

$-121{,}000{,}000 = -1.21 \cdot 10^8$ (Power of ten)

$0.00087 = 8.7 \cdot 10^{-4}$ (Power of ten)

N&0 – 26 **Ordering:** Ordering numbers means placing the numbers in numerical order from the least to the greatest or from the greatest to the least.

Example 26.1:

Order the following numbers from the least to the greatest: 3^3 2.5 $\sqrt{25}$

Note: students may be asked to provide an explanation or place the numbers on a number line.

Answer: $2.5, \sqrt{25}, 3^3$

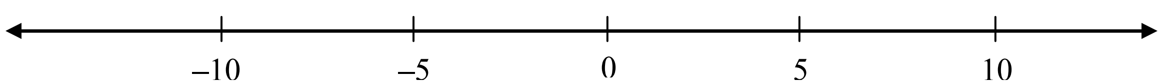
N&0 – 27 **Comparing:** Comparing numbers, *a* and *b*, means to determine if *a* is less than *b*, if *a* is greater than *b*, or if *a* is equal to *b*.

Example 27.1: Alisa is placing $\frac{5}{6}$ on the number line below. Between which two numbers should she place $\frac{5}{6}$?

Answer: $\frac{5}{6}$ should be placed between $\frac{2}{3}$ and 1 because $\frac{5}{6}$ is greater than $\frac{2}{3}$ and less than 1.

N&0 – 28 **Number line:** A number line is a line where every real number corresponds to a unique point on the line. Thus, if two numbers correspond to the same point on the line the numbers are equivalent (See ***N&O – 14***).

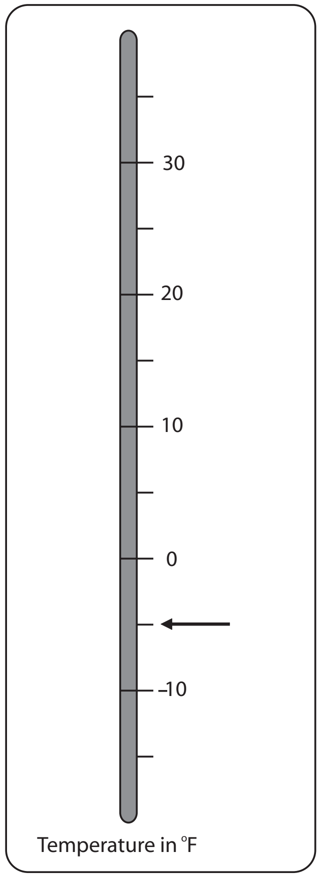
Examples 28.1 – Horizontal number lines:



(Definition *N&O – 28* continued on following page)

Example 28.2 – Using a vertical number line to locate integers in context:

At what temperature is the arrow pointing?



Answer: The arrow is pointing at – 5°F.

Resource Material Prototype

Section 3: Demonstrates Conceptual Understanding of Mathematical Operations

NECAP: M(N&O) – X – 3
Vermont: MX: 3

Note: Appendix A of the Grade-Level Expectations contains the information related to GLEs for grades K – 2.

Definition	Page Number	Definition Number
Describing or illustrating the meaning of a power	28	***N&O – 33***
Effect on magnitude of a whole number when multiplying or dividing by a whole number, fraction, or decimal	29	***N&O – 34***
Inverse relationships in operations	27	***N&O – 29***
Meaning of remainders with respect to division of whole numbers	28	***N&O – 32***
Relationship between repeated addition and multiplication of whole numbers	27	***N&O – 30***
Relationship between repeated subtraction and division of whole numbers	27	***N&O – 31***

N&0 – 29 **Inverse relationships in operations:** Addition and subtraction are inverse operations of each other because addition annuls subtraction and subtraction annuls addition. Similarly, multiplication and division are inverse operations of each other. The chart below shows a few examples.

Operation	Inverse operation
3 **+ 5** = 8	8 **– 5** = 3
8 **– 3** = 5	5 **+ 3** = 8
6 × **7** = 42	42 ÷ **7** = 6
42 ÷ **6 =** 7	7 × **6** = 42

Be advised that there are other inverse operations beyond those shown in the chart above.

N&0 – 30 **Relationship between repeated addition and multiplication of whole numbers:** Multiplication of whole numbers is the same as repeated addition.

Example 30.1: $\mathbf{4} \times 3 = 12$

$$\underbrace{3 + 3 + 3 + 3}_{\mathbf{4}} = 12$$

N&0 – 31 **Relationship between repeated subtraction and division of whole numbers:** Division of whole numbers is the same as repeated subtraction.

Example 31.1: $8 \div 4 = \mathbf{2}$

$$8 \underbrace{- 4 - 4}_{\mathbf{2}} = 0$$

N&0 – 32 **Meaning of remainders with respect to division of whole numbers:** In problem situations involving division of whole numbers, students must decide how to interpret the remainder and then defend their interpretations.

Example 32.1:

One hundred thirty-four students will board school buses to go on a field trip. Each school bus has seats for 60 students. What is the fewest number of buses needed to seat all the students? Explain your answer.

Answer: Three buses will be needed to seat all the students. If you divide 134 students by 60 students per bus, you obtain 2 buses with a remainder of 14 students without seats. So, a third bus is needed to seat (accommodate) the remaining 14 students.

Example 32.2:

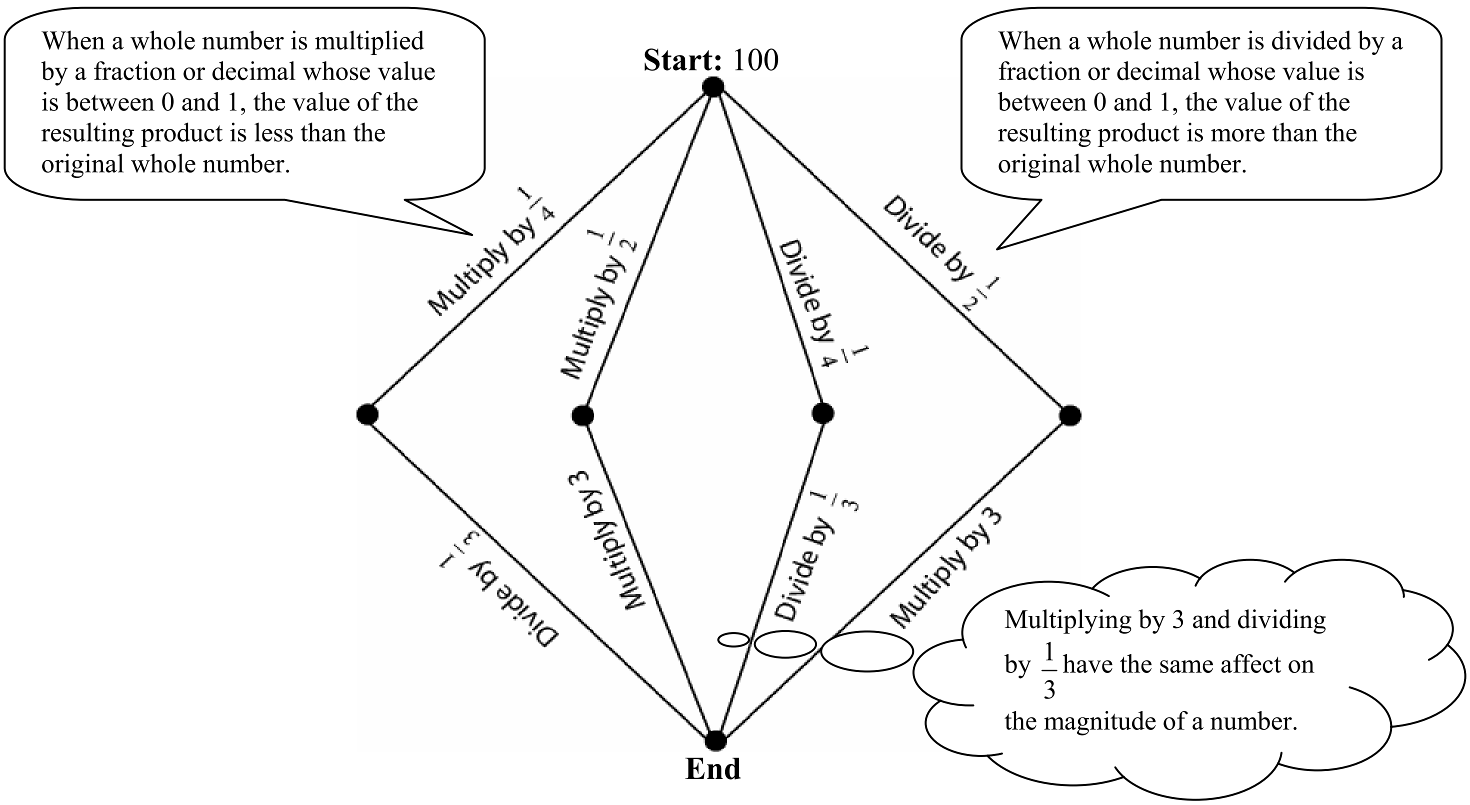
Ms. Thompson has saved $134 to buy some new shoes. She will buy as many pairs of shoes as she can. Each pair of shoes costs $60. How many pairs of shoes can she buy? Explain your answer.

Answer: Ms. Thompson can buy 2 pairs of shoes. If you divide $134 by $60 per pair of shoes, you obtain 2 pairs of shoes with a remainder of $14. Since she can not buy a third pair of shoes with $14, she is restricted to buying only 2 pairs of shoes.

N&0 – 33 **Describing or illustrating the meaning of a power:** Given a base number and an exponent, students will explain how the base number and the exponent (power) are related. (See *N&0 – 24*.)

N&0 – 34 **Effect on magnitude of a whole number when multiplying or dividing by a whole number, fraction, or decimal:** To determine the effect on the magnitude of a whole number when multiplying it by a fraction or decimal means to determine whether the magnitude of the whole number increases, decreases, or stays the same by considering the magnitude of the fraction or decimal.

Example 34.1: The diagram below shows four paths from the **Start** to the **End**. You are given 100 points at the start. Without making any calculations describe which path will result in the largest number of points at the end. Explain your reasoning.



Answer: The route "Divide by $\frac{1}{4}$ and then divide by $\frac{1}{3}$" is the route that would result in the largest number of points at the end. Since dividing by $\frac{1}{3}$ and multiplying by 3 have the same affect on the magnitude of a number, the path that would result in the greatest number can be decided on the first move. In addition, division by a fraction between 0 and 1 results in a larger number than multiplication by a fraction between 0 and 1. Therefore, one of the paths with division by a fraction will result in a larger number than the path with multiplication by a fraction. Since $\frac{1}{4}$ is less than $\frac{1}{2}$, there are more fourths in 100 than halves. (There are 400 fourths in 100, and 200 halves in 100.) Therefore, the path that includes "divide by $\frac{1}{4}$ and then divide by $\frac{1}{3}$" will result in the larger number.

Resource Material Prototype

Section 4: Accurately Solves Problems

NECAP: M(N&O) – X – 4
Vermont: MX: 4

N&O – 35 **Accurately solves problems:** The intent of this GLE is to ensure that students solve problems at various Depth of Knowledge levels (See NECAP Mathematics Test Specifications) by performing accurate calculations (without the use of calculator, manipulatives, or other tools).

Note: As the Depth of Knowledge levels increase the computational demand does not necessarily increase. An attempt is made in the NECAP items to keep the level of computation required at a reasonable level and focus on assessing concepts. Also note, two of the three testing sessions of the NECAP assessment allow the use of calculators. Items where a calculator would take away from the construct being measured (e.g., accurately solves problems) will appear on the session that does not allow calculators, manipulatives, or other tools.

Example 35.1 – (Grade 3) Accurately solves problems involving … addition and subtraction of decimals (in the context of money):

A gallon of milk costs $3.50. A loaf of bread costs $1.59. How much does a gallon of milk and a loaf of bread cost together? Show your work.

Level 1: One-step word problem

Answer: $5.09; $3.50 + $1.59 = $3.00 + $1.00 + $1.00 + $0.09 = $5.09

Example 35.2 – (Grade 4) Accurately solves problems involving … addition and subtraction of decimals:

Use this sign to answer the question below.

Level 2: Two-step problem

Amusement Park Ride Costs	
The first ride	$1.50
Each additional ride	$0.50

How much do 6 rides cost? Show your work.

Item Source: Lager and Petit, Conserving the Mathematical Construct, 2003.

Answer: $4.00; $1.50 + $0.50+ $0.50+ $0.50+ $0.50+ $0.50 = $4.00

(Definition *N&O – 35* continued on following page)

Example 35.3 – (Grade 6) Accurately solves problems involving multiple operations of decimals:

Carolyn and Kim sold 55 cups of lemonade on Monday.
• A cup of lemonade cost $0.10 to make.
• Each cup of lemonade is sold for $0.25.

a. How much did it cost to make 55 cups of lemonade? Show or explain your work.

b. How much money did Carolyn and Kim collect? Show or explain your work.

c. How much profit did Carolyn and Kim collect for selling 55 cups of lemonade? Show or explain your work.

d. Carolyn and Kim will sell lemonade on Wednesday. If the cost to make the lemonade and the price remain the same, what is the least number of cups of lemonade Carolyn and Kim need to sell to collect at least $10.00 **profit**? Show or explain your work.

Level 3: Solves problem with multiple decision points, and planning

Item Source: Adapted from 2002 – 6th grade NHIEAP

Answer:
a. $5.50; 55 × $0.10 = $5.50
b. $13.75; Each group of 4 cups sells for $1.00. 55 ÷ 4 = 13 remainder 3. So, selling 55 cups will make $13.75.
c. $8.25; (55 × $0.25) – (55 × $0.10) = $8.25
d. 67 cups; There are 6 groups of $0.15 in each $1.00 with $0.10 left over. So, in $10.00 there are 60 groups of $0.15 with $1.00 left over. Therefore, 66 cups would be $0.10 less than $10.00 profit. So, they need to sell 67 cups to make at least $10.00 profit.

N&O – 36 **In and out of context:** In and out of context means that some problems will be cast in a context (see Examples 35.1 – 35.3), and others will not be in a context.

Example 36.1 – Non-contextual example:

What is the least common multiple of 12 and 9?

Answer: 36

N&O – 37 **Concept of multiplication:** Solving problems that involve the concept of multiplication means that the problems must demand an understanding of multiplication, not just the application of an algorithm.

Example 37.1:

Mrs. Johnson is arranging 30 chairs in rows. What are all the possible arrangements of the chairs so that:
- there are no fewer than 5 chairs in a row,
- there are no more than 10 chairs in a row, and
- each row has the same number of chairs.

Level 3: Solves problem with multiple decision points, and planning

Answer: 3 rows of 10 chairs, 5 rows of 6 chairs, 6 rows of 5 chairs

N&O – 38 **Factor:** An integer *b* is a factor of a given integer if the product of *b* and some other integer is the given integer (e.g., given the integer 12, 4 is a factor of 12 since $4 \cdot 3 = 12$). A factor is also called a divisor since it divides the given integer evenly (when the given integer is divided by the factor the remainder is 0).

Example 38.1 – Determine whole number factors:

Determine all the whole number factors of 36.

Answer: 1, 2, 3, 4, 6, 9, 12, 18, and 36 are all factors of 36 because each number divides into 36 a whole number of times with no remainder.

Example 38.2 – Determine integer factors:

N&0 – 9 **Integer:** An integer is a number in the set {…, –3, –2, –1, 0, 1, 2, 3, …}.

Determine all the integer factors of 36.

Answer: The integer factors of are –36, –18, –12, –9, –6, –4, –3, –2, –1, 1, 2, 3, 4, 6, 9, 12, 18, 36, because each divides into 36 an integral number of times with no remainder.

At grades K – 6 students will be expected to only generate the whole number factors for a given whole number. For 36, students would only have to generate 1, 2, 3, 4, 6, 9, 12, 18, and 36.

N&O – 39 **Multiples:** A multiple is a number that is the product of a given number and an integer.

Example 39.1: List five multiples of 3.

Sample Answer: 3, 6, 9, 12, and 15 (3 is the given number in this example and each multiple is produced by multiplying 3 by 1, 2, 3, 4, and 5, respectively.)

Example 39.2: List all the multiples of 3.

Answer: The set of multiples of 3 is {…, –15, –12, –9, –6, –3, 0, 3, 6, 9, 12, 15, …}.

Given number	Integer	Product	Multiple
⋮	⋮	⋮	⋮
3	–5	3 × (–5)	–15
3	–4	3 × (–4)	–12
3	–3	3 × (–3)	–9
3	–2	3 × (–2)	–6
3	–1	3 × (–1)	–3
3	0	3 × 0	0
3	1	3 × 1	3
3	2	3 × 2	6
3	3	3 × 3	9
3	4	3 × 4	12
3	5	3 × 5	15
⋮	⋮	⋮	⋮

N&0 – 9 **Integer:** An integer is a number in the set {…, –3, –2, –1, 0, 1, 2, 3, …}.

N&O – 40 **Prime numbers:** A prime number is a whole number greater than 1 that is only divisible by 1 and itself. (Its only factors are 1 and itself.)

Example 40.1:

5 is a prime number because its only factors are 1 and 5.
11 is a prime number because its only factors are 1 and 11.
12 is NOT a prime number because its factors are 1, 2, 3, 4, 6, and 12.

N&O – 41 **Composite number:** A composite number is a number that is not prime (has factors other than 1 and itself).

Example 41.1:

12 is a composite number since its whole number factors are 1, 2, 3, 4, 6, and 12.
2 is NOT a composite number since its only factors are 1 and itself.

N&O – 42 **Greatest Common Factor (GCF):** The greatest common factor of two or more positive integers is the largest factor they have in common.

Example 42.1: What is the greatest common factor (GCF) of 24, 36, and 60?

Number	Factors (Note: Negative factors are not included since we are looking for the greatest common factor.)
24	1, 2, 3, 4, 6, 8, **12**, 24
36	1, 2, 3, 4, 6, 9, **12**, 18, 36
60	1, 2, 3, 4, 5, 6, 10, **12**, 15, 20, 30, 60

Answer: Though there are many common factors of 24, 36, and 60, the greatest common factor (GCF) of 24, 36, and 60 is **12**.

N&O – 43 **Least Common Multiple (LCM):** The least common multiple of two or more positive integers is the smallest positive multiple that they have in common.

Example 43.1: What is the least common multiple (LCM) of 9, 12, and 18?

Number	Positive Multiples
9	9, 18, 27, **36**, 45, 54, 63, 72, 81, …
12	12, 24, **36**, 48, 60, 72, 84, 96, …
18	18, **36**, 54, 72, 90, 108, 126, 144, …

Answer: The least common multiple of 9, 12, and 18 is **36**.

N&O – 44 **Proportional reasoning:** Solving problems involving proportional reasoning means to use proportional reasoning in problem solving situations that may involve ratios, proportions, rates, slope, scale, similarity, percents, probability, and others. It is assumed that throughout instruction students have sufficient opportunities to connect each of these situations to proportional reasoning. (e.g., Students should realize that proportional relationships are described by linear functions of the form $y = kx$).

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